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THEORITICAL AND NUMERICAL ANALYSIS OF CENTRAL CRACK PLATE WITH DIFFERENT ORIENTATION UNDER TENSILE LOAD

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ABSTRACT

Finite element analysis software used to calculate the Stress Intensity Factors, KI and KII, for a central crack in a plate subjected to uniform tensile load for different crack lengths and orientations. Also for inclined crack the SIFs of kinked crack was investigated. For acceptance of FEM model used in computation analysis, Numerical results were compared with theoretical results which getting by solutions of selected equations and good agreement had been found between them. The present study shows that the main important role affects on stress intensity factors is the inclination crack angle (β). For kinked crack, both of Mode I & Mode II of SIFs are strongly depend on the value of (β + α) and there is no effect found when one of them (β or α) change. Furthermore maximum value of Mode II of SIF of kinked crack is found at about [(β + α) = (50o – 60o)].

KEYWORDS : SIFs, Inclined Crack, Kinked Crack, Kinked Angle

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Original Article

INTRODUCTION

The presence of cracks may weaken the material such that fracture occurs at stress much less than the yield or ultimate strength. Fracture mechanics is the methodology used to aid in selecting materials and designed components to minimize the possibility of fracture from cracks. Fracture mechanics is based on the assumption that all engineering materials contain cracks from which failure starts. Cracks lead to high stress concentration near the crack tip; this point should receive particular attention since it is here further crack growth takes place.

There are three modes of loading can be applied to a crack. These load types are categorized as; Mode I (opening mode), Mode II (sliding mode, in – plane shear) and Mode III (tearing mode, antiplane shear). The most critical mode is Mode I because the crack tip carries all the stress whereas in another two modes (Modes II and III) some of the stress is carried by interaction of the opposing crack faces, Thus Mode I is the most common load type encountered in engineering design.

Irwin [1] proposed the description of the stress field at a head of a crack tip by means of only one parameters that called *stress intensity factor*, *K*, which could uniquely define the stress state at the crack tip, without the need to determine the actual stress components (σ_{xx}, σ_{yy} σ_{xy}). Thus;

$$K = \sigma \sqrt{\pi a} \qquad (1)$$

But on otherside, Equation (1) is for the special case of idealized crack in an infinite plate. So that and because of real cracks are affected by the geometry of the component, the applied load stress field and others factors, equation (1) can be generalized as : -

$$K = Y\sigma\sqrt{\pi a} \tag{2}$$

Where *Y is geometrical correction factor which maintain the uniqueness of the stress intensity factor by accounting for the particular geometry.*

Central cracked finite plate with and without kinked is a common specimen in research and practice for fracture mechanics. It has been studied by M.A. Meggiolaro *et. al* [2] who used specialized finite element (FE) and fatigue assessment software to explain fatigue crack and crack growth retardation mechanisms. They obtained the crack path and associated stress intensity factors (SIF) of crack and kinked cracks for several crack lengths and angles. Scott A. Fawaz [3] have been used the well – known Newman / Raju, K solution to predict fatigue life of unsymmetric corner cracks at a hole subjected to tension, bending and bearing load. Also, M. Y. He *et. al* [4] studied the mixed mode intensity factor distributions at the edge of semi – circular and semi – elliptical surface cracks aligned perpendicular to the surface solid subjected to remote shear parallel to the plane of crack. Andrea Spagnoli *et. al* [5] discussed the influence of the degree of crack deflection on the fatigue behavior by using a theoretical model of a periodically – kinked crack. By using the Mellin transform technique, P. S. Theocaris, G. N. Makrakis [6], studied the stress intensity factors at a kinked crack – tip. Ameen Ahmed Nassar [7], used new crack extension technique to evaluate critical stress intensity factor (K_{IC}) for cracked plate in tension for different crack configuration.

Zheyuan Hum *et. al* [8] investigated the effect of crack orientation and boundary condition on mixed – mode SIFs of finite plates with one and two collinear cracks by using boundary collocation method (BCM). R. K. Bhafgat, *et. al* [9] have been studied the effect of crack inclination angle on fracture parameters such as, stress intensity factor for modes (I and II) problem using FEM. P. Baud, *et. al* [10] proposed a scheme to compute the interaction effects between two randomly oriented crack by using an iterative technique based on the method of successive approximations.

By using a crack surface displacement extrapolation technique, Nathera Abdual Hassan Saleh [11] calculated Mode II of stress intensity factor of several crack configuration in plate under uniaxial compression. A rectangular plate with inclined cracks of different crack lengths at different crack inclination angles under biaxial loading condition are analyzed by Rahul Kumar Bhagat *et. al* [12] in mixed mode condition using finite element methode (FEM) to determined the stress intensity factor.

MATERIALS AND METHODS

Figure 1 illustrates the geometry of investigated plate with central cracked. Plate is made up of carbon steel with young's modulus of (200 GPa), poisson's ratio of (0.28) and density of (7820 Kg/ m^3) [13]. It subjected to uniform normal stress (σ). Considering homogeneous and isotropic material, square thin central cracked plate is used in finite element modeling. The length and the width of the plate are (2h = 2w = 200 mm). The crack length is (2a). For inclined crack, the crack angle is (β) whereas the kinked angle is (α) as shown in Figure 2.

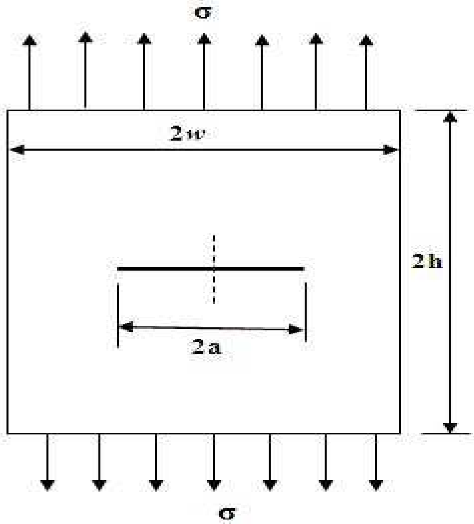


Figure 1: Geometry of Central Cracked

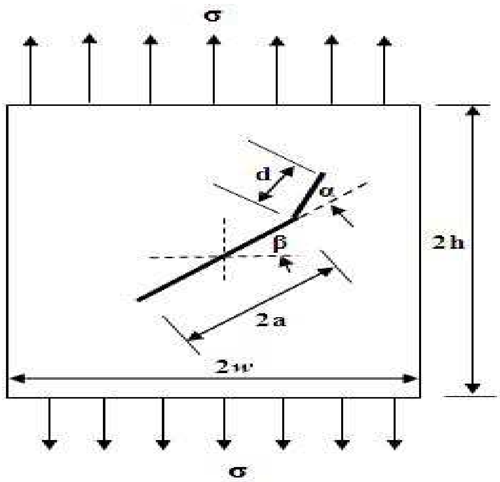


Figure 2: Geometry of Inclined Central Cracked Plate with Dimension Plate with Kinked Cracked

To compute the required results, PLANE 183 element which is shown in Figure 3, has been used in the analysis. The solid modeling is used to generate the 2D model and mesh. It provides more accurate results for mixed (quadrilateral – triangle) automatic meshes and can tolerate irregular shapes without much losing of accuracy.

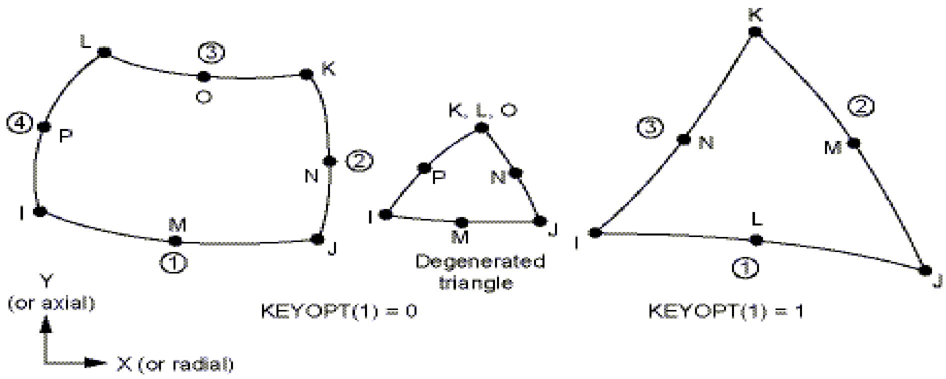
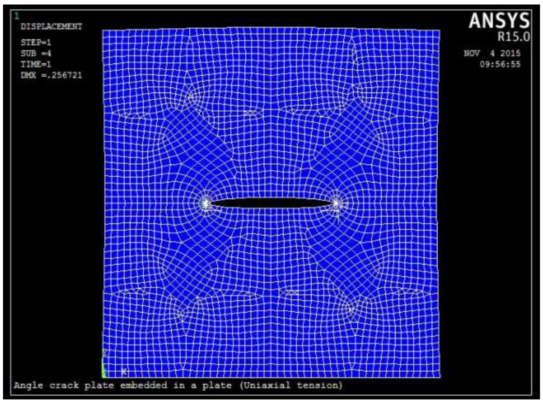


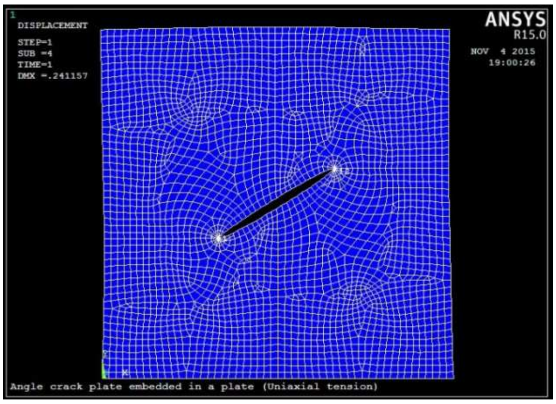
Figure 3: PLANE183 Element Geometry, Coordinate System and Node Locations [14]

Furthermore, a deformed shape of selected Ansys's models for central crack with /without kinked crack at

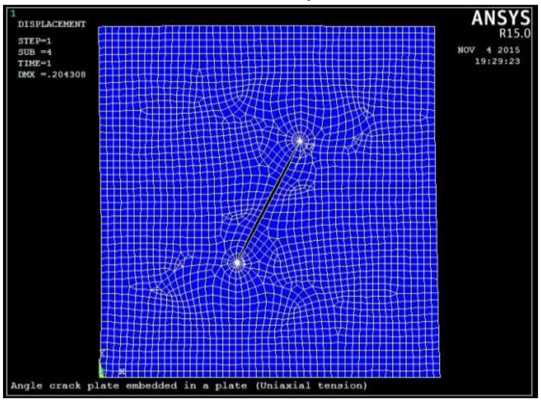
different crack angle (β) and kinked angle (α) are shown in Figure 4.



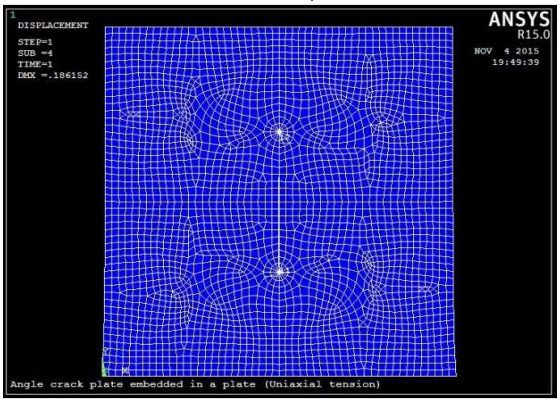
2a = 80 mm, β = 0°



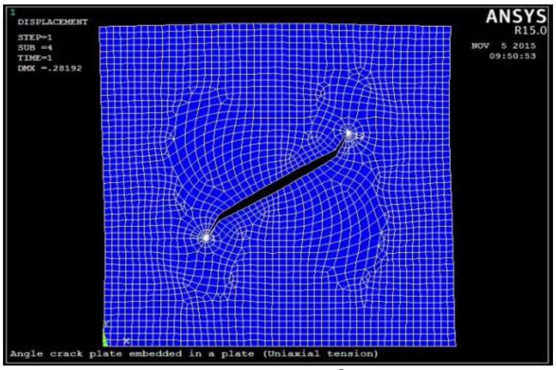
2a = 80 mm, β = 30°



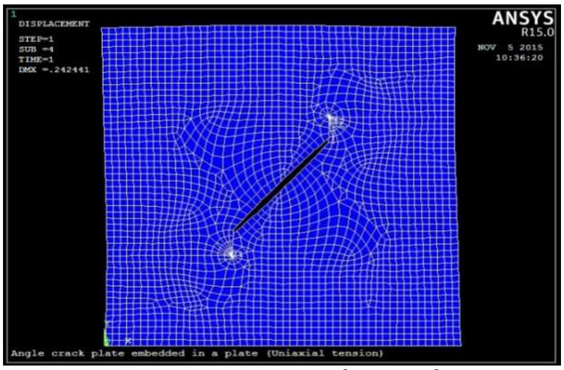
2a = 80 mm, β = 60°



2a = 80 mm, β = 90°



2a = 80 mm, β = 30°, α = 30



2a = 80 mm, β = 45°, α = 45°

Figure 4: Deformed Shape of Selected Anysis's Models for Central Crack with/without Kinked at Different Crack Angle (β) and Kinked Angle (α)

THEORITICAL CALCULATION

Among others, We selected the following two different equations for theoretical calculation of central crack and to compare their results with results have got by *Numerical Method*. The selected Equations are [7, 15]: -

$$K_I = \sigma \sqrt{\pi a}\left(1 + 0.043\left(\frac{a}{w}\right) - 0.491\left(\frac{a}{w}\right)^2 + 7.125\left(\frac{a}{w}\right)^3 - 28.403\left(\frac{a}{w}\right)^4 + 59.583\left(\frac{a}{w}\right)^5 - 65.278\left(\frac{a}{w}\right)^6 +\right.$$

$$29.762\left(\frac{a}{w}\right)^7\Big) \qquad (3)\ [15]$$

$$K_I = \sigma\sqrt{\pi a}\left(1 + 0.256\left(\frac{a}{w}\right) - 1.152\left(\frac{a}{w}\right)^2 + 12.2\left(\frac{a}{w}\right)^3\right) \qquad (4)\ [7]$$

Where: K_I is *the Mode I of Stress Intensity factor* (*SIF*), σ is *the applied tensile stress*, a is *the half length of crack*, and w is *the half width of plate*.

When a crack is oriented with an angle (β) from the positive x – axis as shown in Figure 2 stress intensity factors of Mode I & II are as follows [16]:-

$$KI_\beta = KI_{(o)} \cos^2\beta \qquad (5)$$

$$KII_\beta = KI_{(o)} \cos\beta \sin\beta \qquad (6)$$

Where: $KI_{(o)}$ *is the mode I of stress intensity factor when ($\beta = 0^o$) and is taken as constant*.

Whereas for kinked crack with an angle (α) from the original direction of inclined crack (the plane of crack) (see Figure (2), The local Mode I and Mode II stress intensity factors at the tip of kinked crack (SIFs) are written as following below [16] :-

$$K_{I\alpha} = C_{11} K_I + C_{12} K_{II} \qquad (7)$$

$$K_{II\alpha} = C_{21} K_I + C_{22} K_{II} \qquad (8)$$

Where: *$K_{I\alpha}$ and $K_{II\alpha}$ are the stress intensity factors at the tip of the kinked crack and K_I and K_{II} are the stress intensity factors for the main crack (for present investigation $K_I = KI_\beta$ and K_{II} =* KII_β), *and*

$$C_{11} = \frac{3}{4}\cos\left(\frac{\alpha}{2}\right) + \frac{1}{4}\cos\left(\frac{3\alpha}{2}\right) \qquad (9)$$

$$C_{12} = -\frac{3}{4}\left[\sin\left(\frac{\alpha}{2}\right) + \sin\left(\frac{3\alpha}{2}\right)\right] \qquad (10)$$

$$C_{21} = \frac{1}{4}\left[\sin\left(\frac{\alpha}{2}\right) + \sin\left(\frac{3\alpha}{2}\right)\right] \qquad (11)$$

$$C_{22} = \frac{1}{4}\cos\left(\frac{\alpha}{2}\right) + \frac{3}{4}\cos\left(\frac{3\alpha}{2}\right) \qquad (12\ )$$

THE VALIDITY

Validation of the finite element approach with results available in the literatures, experimental results or theoretical calculation results which can get by solutions of equations are the most important for acceptance of the FEM model used in the computation [12]. In the present investigations, FEM results are compared with results getting by solution of equations (3 & 4). From Figures 5 and 6, it is clear that the theoretical calculations of stress intensity factors, K_I & K_{II} of central crack with or / without inclination are very close to results of analytical solutions at different parameters such as (crack length (a / w), applied stress (σ), crack angle (β). A good agreement is found between the two results especially with equation (3). This means, FE modeling using ANSYS software can be used to determine the stress intensity factors (K_I, K_{II}) for any crack configurations.

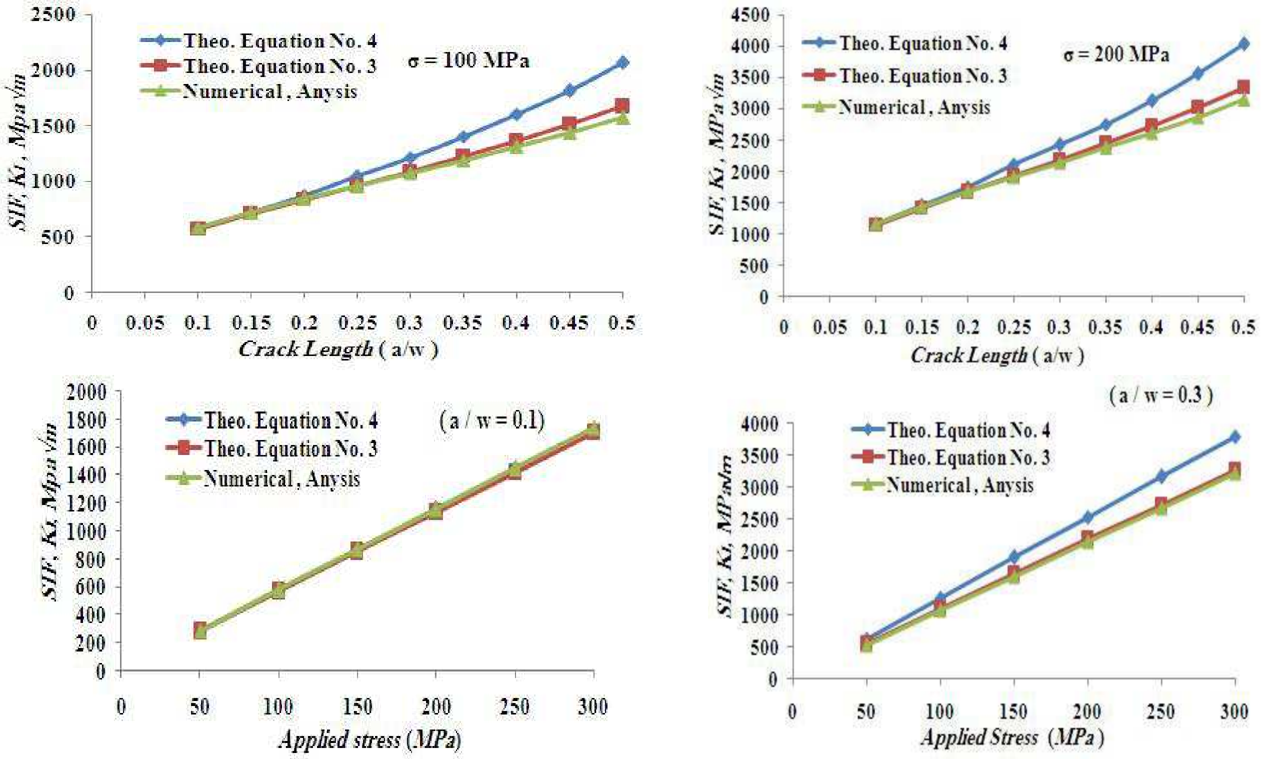


Figure 5: Variation of Theoretical and Numerical SIF, K_1, of Central Crack versus Crack Length (a / w) and Applied Stress

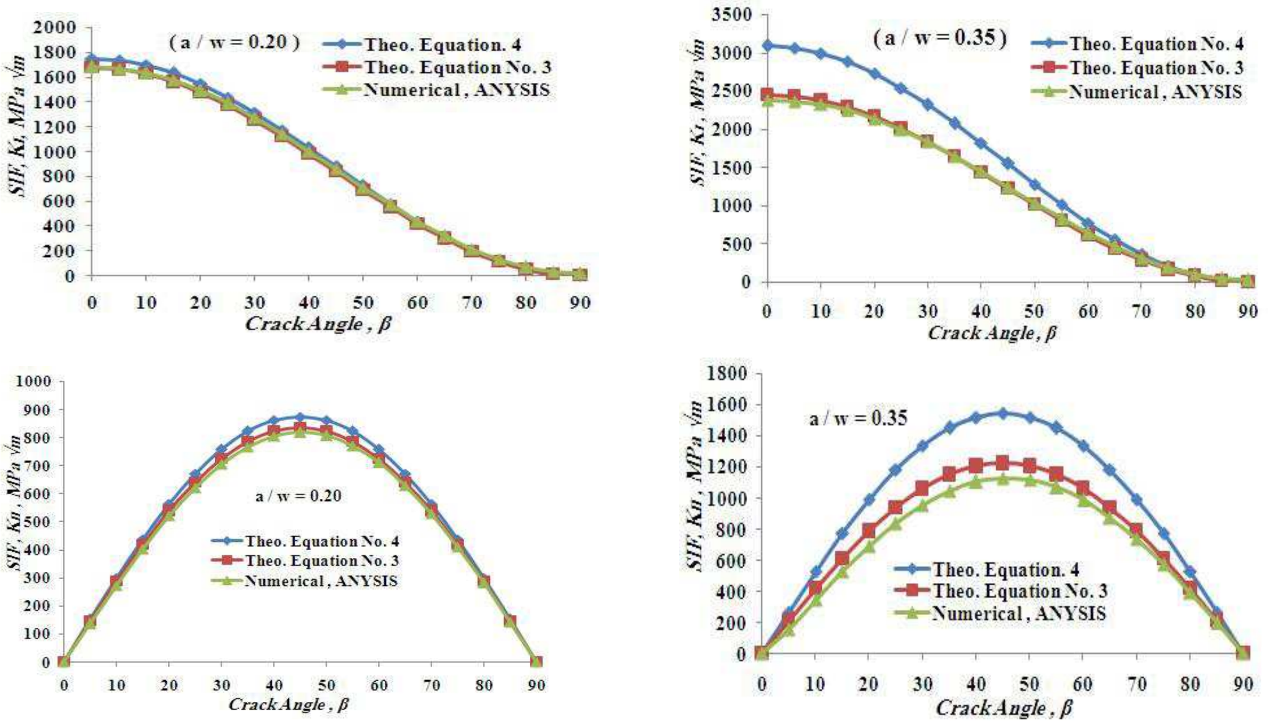


Figure 6. Variation of Theoretical and Numerical SIFs, K_I & K_{II}, of Inclined Central Crack versus Crack Angle (β) at Different Crack Length (a / w).

RESULTS AND DISCUSSIONS

Central Crack Without Kinked

Effect of Applied Stress, Crack Length (a / W) and Crack Angle (β) on SIFs (K_I & K_{II})

As expected, Figure 7 shows, for non - inclined central crack ($\beta = 0^o$), Mode I of stress intensity factor (K_I) increase with an increasing of applied stress and crack length (a / w). When the crack inclined with an angle (β), for all crack length have been investigated, stress intensity factor, K_I, decreases with the increasing of crack angle (β) until reaches to minimum value ($K_I = 0$) at ($\beta = 90^o$), while Mode II, K_{II} increases and reaches it's peak values at about ($\beta = 45^o$) then decreases with an increasing of crack angle (β) as shown. Also, it's clearly seen that, the lower the crack length (a/w) the lower of SIFs (K_I & K_{II}). This observations are agree with the results of [9, 12].

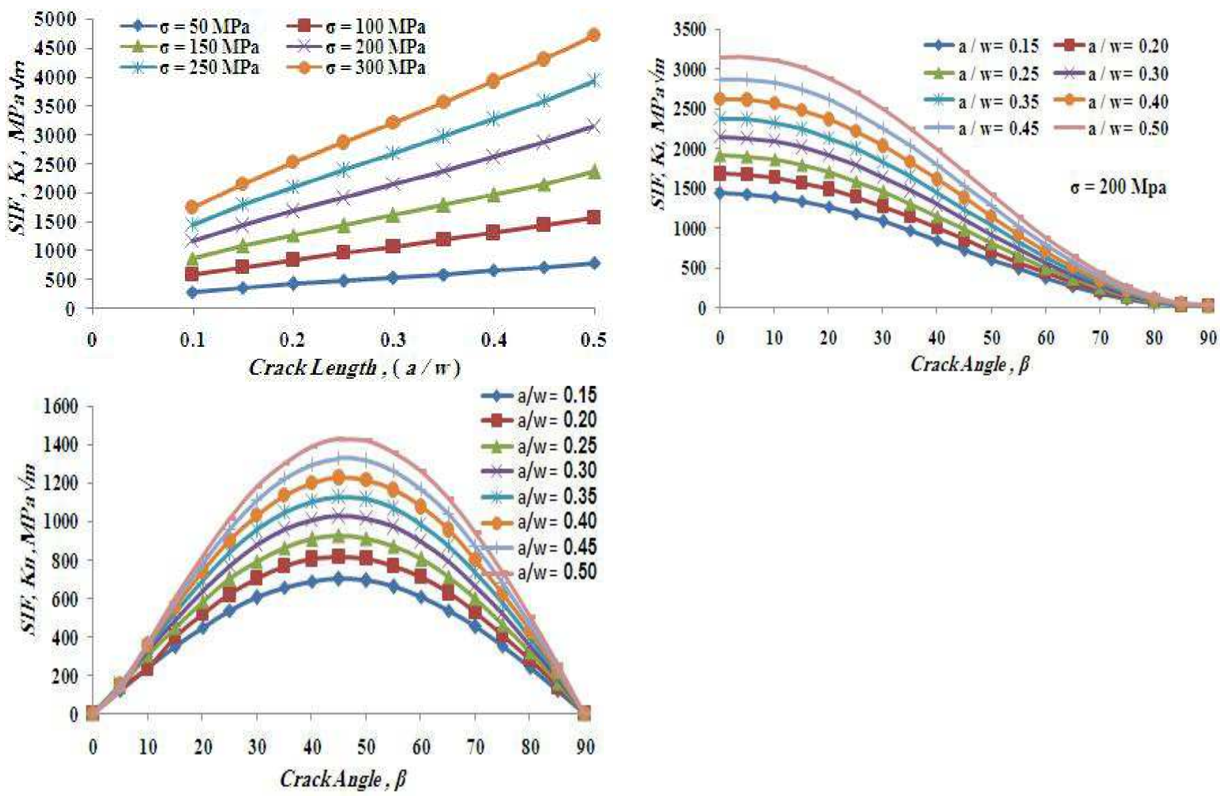


Figure 7: Variation of Numerical SIFs, K_I & K_{II}, versus Crack Angle (β) at Different (a / w)

The Relationship between (K_I & K_{II}) of Central Crack

By comparing the values of K_I & K_{II} at different crack angle (β) as shown in Figures 8, at $\beta \leq 45^o$ the values of stress intensity factors, K_I, are bigger than K_{II}'s values for all crack length (a/w) and this differences in value decrease with an increasing of crack angle (β) even reaches zero at ($\beta = 45^o$), whereas When $\beta \geq 45^o$ the relation between K_I & K_{II} values is reversed (K_{II}'s values is bigger than K_I's values). It means, at $\beta \leq 45^o$, the effect of Mode I, K_I is the dominant, while when $\beta \geq 45^o$ effect of K_{II} is dominated. Therefore both of K_I & K_{II} have to take into account in design according to the expected inclination of the crack. This result is supplement with results of [17].

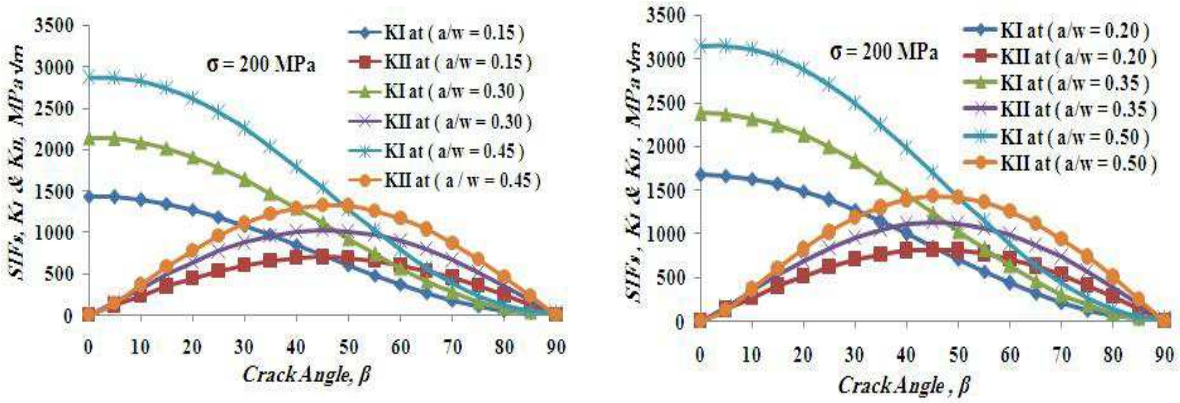


Figure 8: The Relationship between Mode I & Mode II of Stress Intensity Factors at Different Crack Angle (β) and Crack Length (a / w)

Central Crack with Kinked

Effect of crack angle (β) and kinked crack angle (α) on SIFs ($K_{I(\alpha)}$ & $K_{II(\alpha)}$)

From Figure 9, Mode I of SIF of kinked crack, $K_{I(\alpha)}$, decreases with the increasing of both crack angle (β) and kinked crack angle (α) for all crack length (a/w), and the value of $K_{I(\alpha)}$ equal to zero when (β + α) about equal to (85^o – 90^o). It is clear that, for all crack length, when crack angle (β) increases, SIF, $K_{I(\alpha)}$ reaches zero value at lower kinked crack angle (α), So that, the effect of crack angle (β) and kinked crack angle (α) on SIF, $K_{I(\alpha)}$ can be summarized as : ***For any value of $K_{I(\alpha)}$, If crack angle (β) increases, kinked crack angle (α) decreases.*** It is an inverse relationship between crack (β) and kinked crack angle (α).

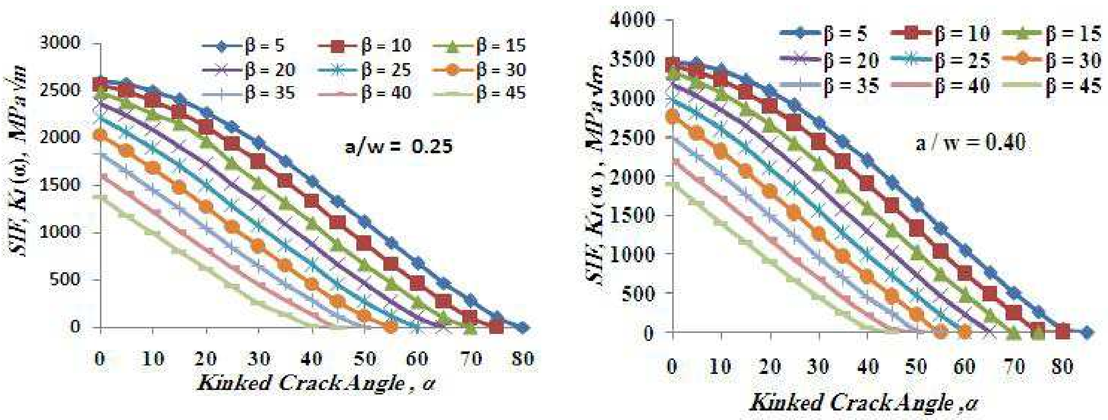


Figure 9: Variation of Mode I of Stress Intensity Factors, $K_{I(\alpha)}$, versus Kinked Crack Angle (α) at Different crack Angle (β) and Crack Length (a / w).

$K_{II(\alpha)}$ takes the same behaviour of Mode II, SIF of the main crack, $K_{II(\beta)}$, where increases with increasing both of crack and kinked angles (β & α) until reaches it's maximum value at (β + α) equal about (55^o – 60^o) then decreases as shown in Figure 10. It means, the maximum value of Mode II of SIF of kinked crack $K_{II(\alpha)}$ doesn't occur at (α = 45^o) as had been found for main crack, $K_{II(\beta)}$ (see figures 6 & 7), but also it depends on the value of crack angle (β). It is strongly depends on the summation of (β & α).

In addition to that, the main interesting observation which can be seen is ; at (β + α) < (55^o – 60^o), the higher value of crack angle (β), the higher value of $K_{II(\alpha)}$ is, while at (β + α) > (55^o – 60^o), the $K_{II(\alpha)}$ value reverses where the higher value of crack angle (β), the lower value of $K_{II(\alpha)}$ is.

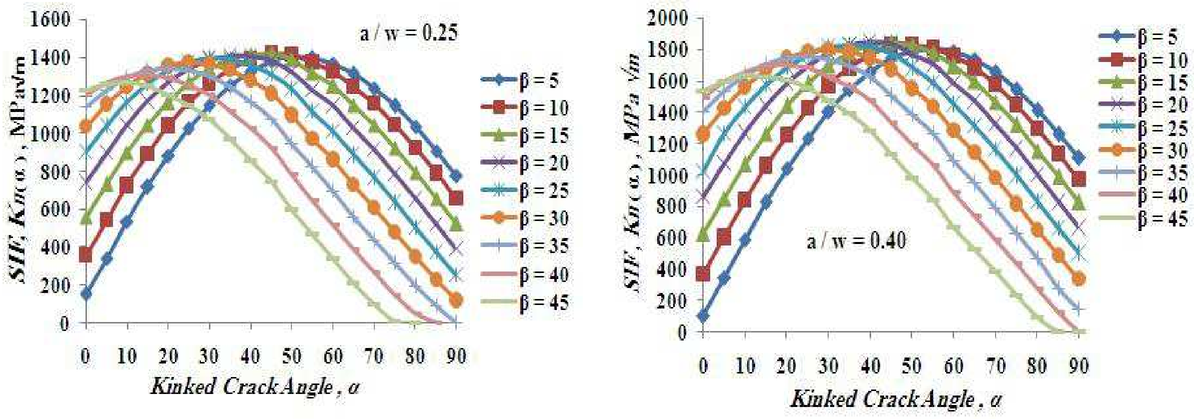


Figure 10: Variation of Mode II of Stress Intensity Factors, $K_{II(\alpha)}$, versus Kinked Crack Angle (α) at different Crack Angle (β) and Crack length (a / w)

On other hand, at constant value of (β + α), there is no effect have been found in values of $K_{I(\alpha)}$ and $K_{II(\alpha)}$ by varying the values of one of (β or α) angles, while there is a considerable effect can be seen when (β + α) change together as shown in figure 11.

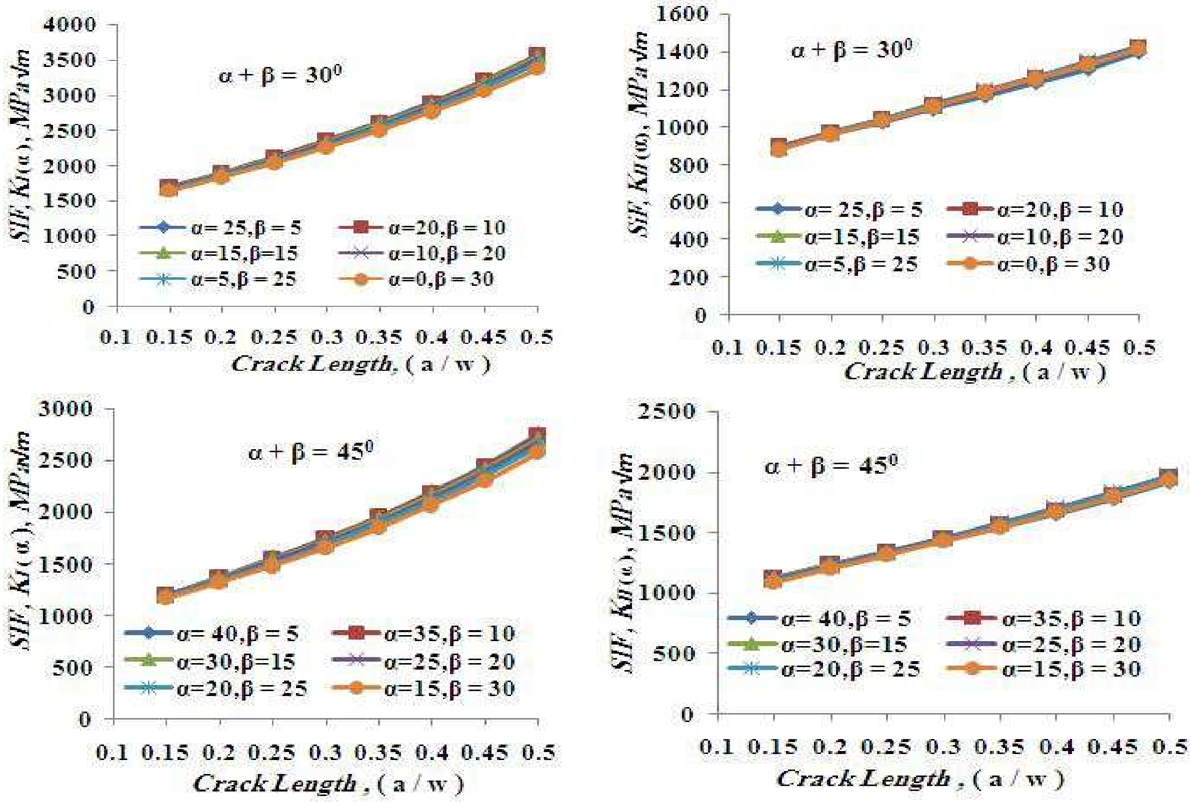


Figure 11: Effect of Change Crack Angle (β) or Kinked Angle (α) on Stress Intensity Factors, $K_{I(\alpha)}$ & $K_{II(\alpha)}$ at Constant (β + α)

The Relationship between SIFs ($K_{I(\alpha)}$ & $K_{II(\alpha)}$) of Kinked Crack

Figure 12 clearly shows the relationship between Mode I & Mode II of SIFs of kinked crack, $K_{I(\alpha)}$ & $K_{II(\alpha)}$. The same relationship between $K_{I(\beta)}$ & $K_{II(\beta)}$ is found. It can be seen, ; at ($\beta + \alpha$) < ($55^o - 60^o$), the SIF's value of $K_{I(\alpha)}$ is bigger than the SIF's value of $K_{II(\alpha)}$, whereas when ($\beta + \alpha$) exceeds ($55^o - 60^o$), the SIF's value of $K_{II(\alpha)}$ is bigger.

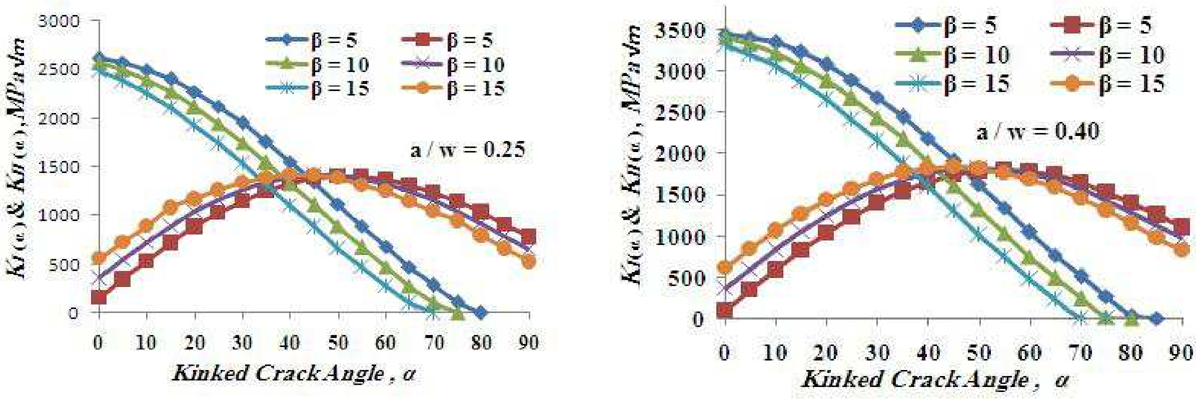
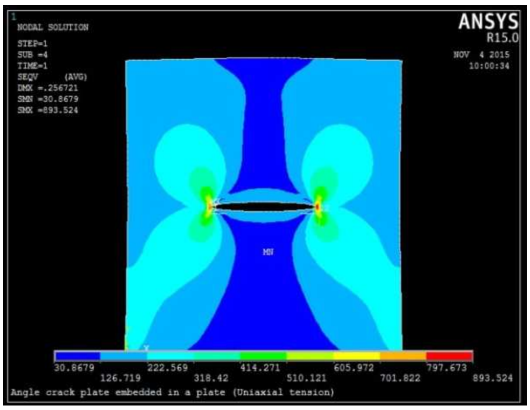
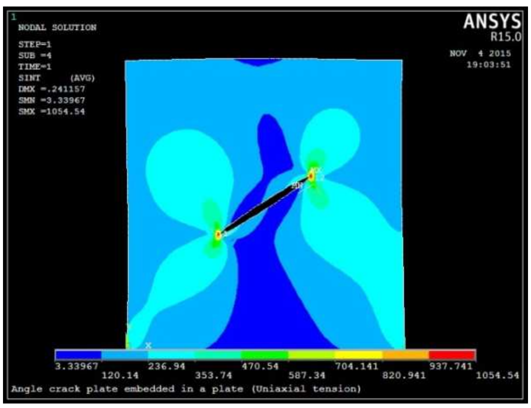


Figure 12: The Relationship between Mode I & Mode II of Stress Intensity Factors of Kinked Rack at Crack Angle (β) and Crack Length (a / w)

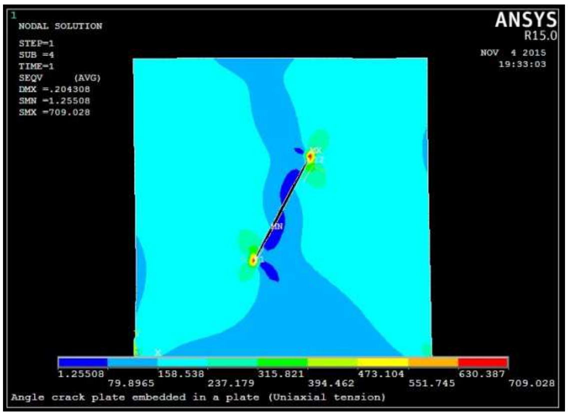
Finally, figure 13 illustrated clearly Von – mises stresses of some of selected investigated Ansys's models for central crack with and without inclination at different crack and kinked angles (β & α). From figures 4 & 13 It can see the effect of crack angle (β) and kinked angle (α) on the stress distribution and structure deformation in investigated plate model.



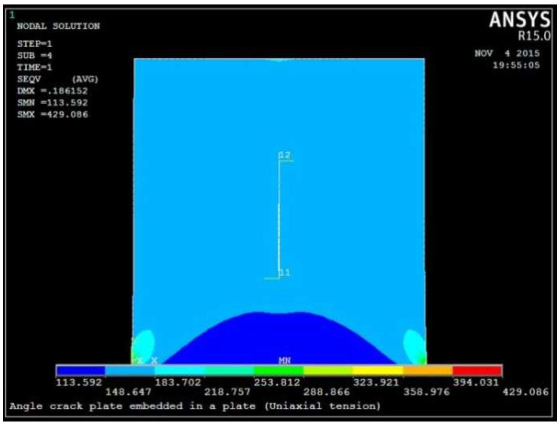
2a = 80 mm, β = 0°



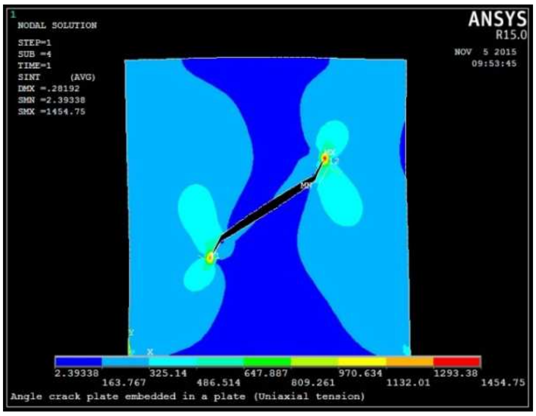
2a = 80 mm, β = 30°



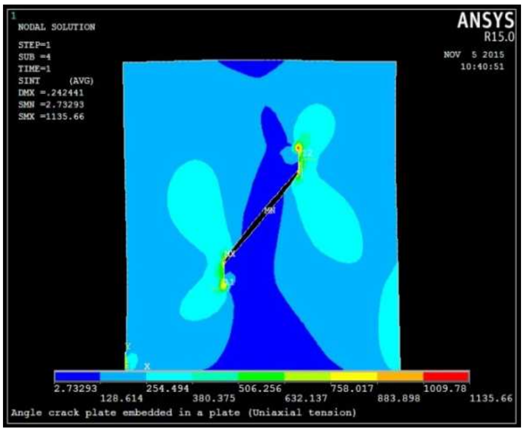
2a = 80 mm, β = 60^o



2a = 80 mm, β = 90^o



2a 80 mm = , α = 30^o, β = 30^o



2a = 80 mm, β = 45^o, α = 45^o

Figure 13: Von–Mises Stresses of Selected ANYSIS'S Models for Central Crack with / without Kinked at different Crack Angle (β) and Kinked Angle (α).

CONCLUSIONS

The results of present investigation can be summarized as :-

- The main important role that affects on Mode I and Mode II of stress intensity factor is the crack angle (β) Where at β < 45^o, the value of Mode I ($K_{I\ (\beta)}$) is bigger than the value of Mode II ($K_{II\ (\beta)}$), whereas when β > 45o, the value of Mode II of SIF ($K_{II\ (\beta)}$ is bigger than the value of Mode I of SIF ($K_{I\ (\beta)}$. Furthermore the maximum value of Mode I ($K_{I\ (\beta)}$) is at β = 0^o while the maximum value of Mode II ($K_{I\ (\beta)}$) at (β = 45^o)
- Both of Modes I & II of SIF of kinked crack, (K_I (α) & K_I (α)), are strongly depend on the value of (β + α), and there is no effect found when one of them (β or α) change.
- Maximum value of Modes II of SIF of kinked crack is found at about [(β + α) = (55^o – 60^o)], but at (β + α) < (55o–60o), the lower value of crack angle (β), the lower value of K_{II} (α) is, whereas at (β + α) > (55o – 60o), the lower value of crack angle (β), the higher value of ($K_{II\ (\alpha)}$) is.

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